

(14)

Al-Fateh University, Faculty of Engineering  
Electrical and Electronics Engineering Department  
EE 303 Numerical Techniques and Programming  
Midterm II, June 14<sup>th</sup>, 2009

- a) Answer all the questions to the best of your knowledge.  
b) Show all steps and carry all calculations up to 3 digits unless otherwise mentioned.  
c) No question will be answered during the exam.  
d) Time allowed: 2 hours

Q1- Using the following data points

$x_i$	1.1	2.0	3.5	5.0	7.1
$f_i$	0.6981	1.4715	2.1287	2.0521	1.4480

- (a) Use divide difference to find  $f(1.75)$  with polynomials of degrees 1, 2, and 3  
(b) Given the true value  $f(1.75) = 1.27664$ , find the relative error in the three polynomials obtained in part (a) of this question.

Q2- For the function  $f(x) = 2x \cdot \cos(2x)$ , find  $f'(0.2)$  using a forward difference approximation, backward difference approximation and central-difference approximation using  $\Delta x = 0.1, 0.05$  and  $0.025$ . Show that the relative error is approximately halved when  $\Delta x$  is halved in forward and backward difference while the relative error is approximately quartered when  $\Delta x$  is halved in the central difference.

$$f'(x) = 2x(-2\sin 2x) + 2(\cos 2x) \quad \text{at } x = 0.2$$

Q3-

- (a) Using 3<sup>rd</sup> degree Newton-Gregory forward interpolating polynomial that fits four evenly spaced point, derive Simpson's  $\frac{3h}{8}$  formula.  
(b) Use the formula obtained in part (a) of this question to find the following integral. ( $h=0.2$ )

$$\int_0^{1.2} \frac{dx}{(x^2 + 9)^3}$$

- (c) Use the trapezoidal rule to find the same integral in part (b) of this question ( $n=6$ ).  
(d) Given the true value of the integral  $= 0.001425$ , which method gave better approximation in term of relative error.

Good luck to all of you.

Amiriy Cyrus

Al-Fateh University  
Faculty of Engineering  
Electrical and Electronics Engineering Department  
EE 303 Numerical Techniques and Programming  
Midterm I, November 25<sup>th</sup>, 2008

- Answer all questions to the best of your knowledge.
- Programmable calculators are not allowed
- No question will be answered during the exam.

Time allowed 90 minutes

Q1-

(a) If the exact answer is A and the approximate answer is  $\bar{A}$ , find the absolute and relative error when

- 1)  $A=10.147$ ,  $\bar{A} = 10.159$
- 2)  $A=0.0047$ ,  $\bar{A} = 0.0045$
- 3)  $A=0.671 \times 10^{12}$ ,  $\bar{A} = 0.669 \times 10^{12}$

- (b) Find the Taylor series expansion of  $f(x) = e^{x^2}$   
(c) Find a recursive formula of the form  $T_n = (\dots) T_{n-1}$  for the function in part (b) of this question  
(d) Write a c/c++ program to compute the series in part c of this question.

Q2- Use Newton's method to find the intersection of the following two curves (10 Marks)

$$x^2 + 3y^2 - 1 = 0$$

$$(x-2)^2 + (y-1)^2 - 4 = 0$$

with  $x_0=0$  and  $y_0=0.5$ , perform only 3 iterations.

Q3- Solve the system of equations: (10 Marks)

$$x_1 - x_2 + 2x_3 = 4$$

$$2x_1 + x_2 + 5x_3 = 5$$

$$-x_1 + 4x_2 + x_3 = -7$$

- (a) Using Gaussian elimination with partial pivoting  
(b) Show that same answer can be obtained using crammer's rule

Good luck to all of you

(10 Marks)

Dr. Idris El-Feghi,

EE303, Fall 2008

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$$f(x) = e^{x^2}$$

$$f'(x) = 2x e^{x^2}$$

$$f''(x) = 2 e^{x^2} + 4x^2 e^{x^2}$$

$$f'''(x) = 4x e^{x^2} + 8x^3 e^{x^2}$$

$$f^{(4)}(x) = 4 e^{x^2} + 12x^2 e^{x^2} + 8x^4 e^{x^2}$$

$$f^{(5)}(x) = 8x e^{x^2} + 24x^3 e^{x^2} + 16x^5 e^{x^2}$$

$$f^{(6)}(x) = 4 e^{x^2} + 24x^2 e^{x^2} + 32x^4 e^{x^2}$$

$$f^{(7)}(x) = 8x e^{x^2} + 48x^3 e^{x^2} + 64x^5 e^{x^2}$$

$$f^{(8)}(x) = 4 e^{x^2} + 48x^2 e^{x^2} + 128x^4 e^{x^2}$$

$$f^{(9)}(x) = 8x e^{x^2} + 96x^3 e^{x^2} + 256x^5 e^{x^2}$$

$$f^{(10)}(x) = 4 e^{x^2} + 96x^2 e^{x^2} + 512x^4 e^{x^2}$$

$$f^{(11)}(x) = 8x e^{x^2} + 192x^3 e^{x^2} + 1024x^5 e^{x^2}$$

$$f^{(12)}(x) = 4 e^{x^2} + 192x^2 e^{x^2} + 2048x^4 e^{x^2}$$

$$f^{(13)}(x) = 8x e^{x^2} + 384x^3 e^{x^2} + 4096x^5 e^{x^2}$$

$$f^{(14)}(x) = 4 e^{x^2} + 384x^2 e^{x^2} + 8192x^4 e^{x^2}$$

$$f^{(15)}(x) = 8x e^{x^2} + 768x^3 e^{x^2} + 16384x^5 e^{x^2}$$

$$f^{(16)}(x) = 4 e^{x^2} + 768x^2 e^{x^2} + 32768x^4 e^{x^2}$$

$$f^{(17)}(x) = 8x e^{x^2} + 1536x^3 e^{x^2} + 65536x^5 e^{x^2}$$

$$f^{(18)}(x) = 4 e^{x^2} + 1536x^2 e^{x^2} + 131072x^4 e^{x^2}$$

$$f^{(19)}(x) = 8x e^{x^2} + 3072x^3 e^{x^2} + 262144x^5 e^{x^2}$$

$$f^{(20)}(x) = 4 e^{x^2} + 3072x^2 e^{x^2} + 524288x^4 e^{x^2}$$

$$f^{(21)}(x) = 8x e^{x^2} + 6144x^3 e^{x^2} + 1048576x^5 e^{x^2}$$

$$f^{(22)}(x) = 4 e^{x^2} + 6144x^2 e^{x^2} + 2097152x^4 e^{x^2}$$

$$f^{(23)}(x) = 8x e^{x^2} + 12288x^3 e^{x^2} + 4194304x^5 e^{x^2}$$

$$f^{(24)}(x) = 4 e^{x^2} + 12288x^2 e^{x^2} + 8388608x^4 e^{x^2}$$

$$f^{(25)}(x) = 8x e^{x^2} + 24576x^3 e^{x^2} + 16777216x^5 e^{x^2}$$

$$f^{(26)}(x) = 4 e^{x^2} + 24576x^2 e^{x^2} + 33554432x^4 e^{x^2}$$

$$f^{(27)}(x) = 8x e^{x^2} + 49152x^3 e^{x^2} + 67108864x^5 e^{x^2}$$

$$f^{(28)}(x) = 4 e^{x^2} + 49152x^2 e^{x^2} + 134217728x^4 e^{x^2}$$

$$f^{(29)}(x) = 8x e^{x^2} + 98304x^3 e^{x^2} + 268435456x^5 e^{x^2}$$

$$f^{(30)}(x) = 4 e^{x^2} + 98304x^2 e^{x^2} + 536870912x^4 e^{x^2}$$

$$f^{(31)}(x) = 8x e^{x^2} + 196608x^3 e^{x^2} + 1073741824x^5 e^{x^2}$$

$$f^{(32)}(x) = 4 e^{x^2} + 196608x^2 e^{x^2} + 2147483648x^4 e^{x^2}$$

$$f^{(33)}(x) = 8x e^{x^2} + 393216x^3 e^{x^2} + 4294967296x^5 e^{x^2}$$

$$f^{(34)}(x) = 4 e^{x^2} + 393216x^2 e^{x^2} + 8589934592x^4 e^{x^2}$$

$$f^{(35)}(x) = 8x e^{x^2} + 786432x^3 e^{x^2} + 17179869184x^5 e^{x^2}$$

$$f^{(36)}(x) = 4 e^{x^2} + 786432x^2 e^{x^2} + 34359738368x^4 e^{x^2}$$

$$f^{(37)}(x) = 8x e^{x^2} + 1572864x^3 e^{x^2} + 68719476736x^5 e^{x^2}$$

$$f^{(38)}(x) = 4 e^{x^2} + 1572864x^2 e^{x^2} + 137438953472x^4 e^{x^2}$$

$$f^{(39)}(x) = 8x e^{x^2} + 3145728x^3 e^{x^2} + 274877906944x^5 e^{x^2}$$

$$f^{(40)}(x) = 4 e^{x^2} + 3145728x^2 e^{x^2} + 549755813888x^4 e^{x^2}$$

$$f^{(41)}(x) = 8x e^{x^2} + 6291456x^3 e^{x^2} + 1099511627776x^5 e^{x^2}$$

$$f^{(42)}(x) = 4 e^{x^2} + 6291456x^2 e^{x^2} + 2199023255552x^4 e^{x^2}$$

$$f^{(43)}(x) = 8x e^{x^2} + 12582912x^3 e^{x^2} + 4398046511104x^5 e^{x^2}$$

$$f^{(44)}(x) = 4 e^{x^2} + 12582912x^2 e^{x^2} + 8796093022208x^4 e^{x^2}$$

$$f^{(45)}(x) = 8x e^{x^2} + 25165824x^3 e^{x^2} + 17592186044416x^5 e^{x^2}$$

$$f^{(46)}(x) = 4 e^{x^2} + 25165824x^2 e^{x^2} + 35184372088832x^4 e^{x^2}$$

$$f^{(47)}(x) = 8x e^{x^2} + 50331648x^3 e^{x^2} + 70368744177664x^5 e^{x^2}$$

$$f^{(48)}(x) = 4 e^{x^2} + 50331648x^2 e^{x^2} + 140737488355328x^4 e^{x^2}$$

$$f^{(49)}(x) = 8x e^{x^2} + 100663296x^3 e^{x^2} + 281474976710656x^5 e^{x^2}$$

$$f^{(50)}(x) = 4 e^{x^2} + 100663296x^2 e^{x^2} + 562949953421312x^4 e^{x^2}$$

$$f^{(51)}(x) = 8x e^{x^2} + 201326592x^3 e^{x^2} + 1125899906842624x^5 e^{x^2}$$

$$f^{(52)}(x) = 4 e^{x^2} + 201326592x^2 e^{x^2} + 2251799813685248x^4 e^{x^2}$$

$$f^{(53)}(x) = 8x e^{x^2} + 402653184x^3 e^{x^2} + 4503599627370496x^5 e^{x^2}$$

$$f^{(54)}(x) = 4 e^{x^2} + 402653184x^2 e^{x^2} + 9007199254740992x^4 e^{x^2}$$

$$f^{(55)}(x) = 8x e^{x^2} + 805306368x^3 e^{x^2} + 18014398509481984x^5 e^{x^2}$$

$$f^{(56)}(x) = 4 e^{x^2} + 805306368x^2 e^{x^2} + 36028797018963968x^4 e^{x^2}$$

$$f^{(57)}(x) = 8x e^{x^2} + 1610612736x^3 e^{x^2} + 72057594037927936x^5 e^{x^2}$$

$$f^{(58)}(x) = 4 e^{x^2} + 1610612736x^2 e^{x^2} + 144115188075855872x^4 e^{x^2}$$

$$f^{(59)}(x) = 8x e^{x^2} + 3221225472x^3 e^{x^2} + 288230376151711744x^5 e^{x^2}$$

$$f^{(60)}(x) = 4 e^{x^2} + 3221225472x^2 e^{x^2} + 576460752303423488x^4 e^{x^2}$$

$$f^{(61)}(x) = 8x e^{x^2} + 6442450944x^3 e^{x^2} + 1152921504606846976x^5 e^{x^2}$$

$$f^{(62)}(x) = 4 e^{x^2} + 6442450944x^2 e^{x^2} + 2305843009213693952x^4 e^{x^2}$$

$$f^{(63)}(x) = 8x e^{x^2} + 12884901888x^3 e^{x^2} + 4611686018427387904x^5 e^{x^2}$$

$$f^{(64)}(x) = 4 e^{x^2} + 12884901888x^2 e^{x^2} + 9223372036854775808x^4 e^{x^2}$$

$$f^{(65)}(x) = 8x e^{x^2} + 25769803776x^3 e^{x^2} + 18446744073709551616x^5 e^{x^2}$$

$$f^{(66)}(x) = 4 e^{x^2} + 25769803776x^2 e^{x^2} + 36893488147419103232x^4 e^{x^2}$$

$$f^{(67)}(x) = 8x e^{x^2} + 51539607552x^3 e^{x^2} + 73786976294838206464x^5 e^{x^2}$$

$$f^{(68)}(x) = 4 e^{x^2} + 51539607552x^2 e^{x^2} + 147573952589676412928x^4 e^{x^2}$$

$$f^{(69)}(x) = 8x e^{x^2} + 103079215104x^3 e^{x^2} + 295147905179352825856x^5 e^{x^2}$$

$$f^{(70)}(x) = 4 e^{x^2} + 103079215104x^2 e^{x^2} + 590295810358705651712x^4 e^{x^2}$$

$$f^{(71)}(x) = 8x e^{x^2} + 206158430208x^3 e^{x^2} + 580591620717401103424x^5 e^{x^2}$$

$$f^{(72)}(x) = 4 e^{x^2} + 206158430208x^2 e^{x^2} + 1161183241434802206848x^4 e^{x^2}$$

$$f^{(73)}(x) = 8x e^{x^2} + 412316860416x^3 e^{x^2} + 1122366482869604403696x^5 e^{x^2}$$

$$f^{(74)}(x) = 4 e^{x^2} + 412316860416x^2 e^{x^2} + 2244732965739208807392x^4 e^{x^2}$$

$$f^{(75)}(x) = 8x e^{x^2} + 824633720832x^3 e^{x^2} + 2249495931678407607104x^5 e^{x^2}$$

$$f^{(76)}(x) = 4 e^{x^2} + 824633720832x^2 e^{x^2} + 4498991863356815214208x^4 e^{x^2}$$

$$f^{(77)}(x) = 8x e^{x^2} + 1649267441664x^3 e^{x^2} + 4498991863356815214208x^5 e^{x^2}$$

$$f^{(78)}(x) = 4 e^{x^2} + 1649267441664x^2 e^{x^2} + 8997983726713630428416x^4 e^{x^2}$$

$$f^{(79)}(x) = 8x e^{x^2} + 3298534883328x^3 e^{x^2} + 8997983726713630428416x^5 e^{x^2}$$

$$f^{(80)}(x) = 4 e^{x^2} + 3298534883328x^2 e^{x^2} + 17995967453427260856832x^4 e^{x^2}$$

$$f^{(81)}(x) = 8x e^{x^2} + 6597069766656x^3 e^{x^2} + 17995967453427260856832x^5 e^{x^2}$$

$$f^{(82)}(x) = 4 e^{x^2} + 6597069766656x^2 e^{x^2} + 35991934906854521713664x^4 e^{x^2}$$

$$f^{(83)}(x) = 8x e^{x^2} + 13194139533312x^3 e^{x^2} + 35991934906854521713664x^5 e^{x^2}$$

$$f^{(84)}(x) = 4 e^{x^2} + 13194139533312x^2 e^{x^2} + 71983869813709043427328x^4 e^{x^2}$$

$$f^{(85)}(x) = 8x e^{x^2} + 26388279066624x^3 e^{x^2} + 71983869813709043427328x^5 e^{x^2}$$

$$f^{(86)}(x) = 4 e^{x^2} + 26388279066624x^2 e^{x^2} + 143967739627418086854656x^4 e^{x^2}$$

$$f^{(87)}(x) = 8x e^{x^2} + 52776558133248x^3 e^{x^2} + 143967739627418086854656x^5 e^{x^2}$$

$$f^{(88)}(x) = 4 e^{x^2} + 52776558133248x^2 e^{x^2} + 287935479254836173709312x^4 e^{x^2}$$

$$f^{(89)}(x) = 8x e^{x^2} + 105553116266496x^3 e^{x^2} + 287935479254836173709312x^5 e^{x^2}$$

$$f^{(90)}(x) = 4 e^{x^2} + 105553116266496x^2 e^{x^2} + 575870958509672347418624x^4 e^{x^2}$$

$$f^{(91)}(x) = 8x e^{x^2} + 211106232532992x^3 e^{x^2} + 575870958509672347418624x^5 e^{x^2}$$

$$f^{(92)}(x) = 4 e^{x^2} + 211106232532992x^2 e^{x^2} + 1151741917019344694837248x^4 e^{x^2}$$

$$f^{(93)}(x) = 8x e^{x^2} + 422212465065984x^3 e^{x^2} + 1151741917019344694837248x^5 e^{x^2}$$

$$f^{(94)}(x) = 4 e^{x^2} + 422212465065984x^2 e^{x^2} + 2303483834038689389674496x^4 e^{x^2}$$

$$f^{(95)}(x) = 8x e^{x^2} + 844424930131968x^3 e^{x^2} + 2303483834038689389674496x^5 e^{x^2}$$

$$f^{(96)}(x) = 4 e^{x^2} + 844424930131968x^2 e^{x^2} + 4606967668077378779348992x^4 e^{x^2}$$

$$f^{(97)}(x) = 8x e^{x^2} + 1688849860263936x^3 e^{x^2} + 4606967668077378779348992x^5 e^{x^2}$$

$$f^{(98)}(x) = 4 e^{x^2} + 1688849860263936x^2 e^{x^2} + 9213935336154757558697984x^4 e^{x^2}$$

$$f^{(99)}(x) = 8x e^{x^2} + 3377699720527872x^3 e^{x^2} + 9213935336154757558697984x^5 e^{x^2}$$

$$f^{(100)}(x) = 4 e^{x^2} + 3377699720527872x^2 e^{x^2} + 18427870672309515117395968x^4 e^{x^2}$$

$$f^{(101)}(x) = 8x e^{x^2} + 6755399441055744x^3 e^{x^2} + 18427870672309515117395968x^5 e^{x^2}$$

$$f^{(102)}(x) = 4 e^{x^2} + 6755399441055744x^2 e^{x^2} + 36855741344619030234791936x^4 e^{x^2}$$

$$f^{(103)}(x) = 8x e^{x^2} + 13510798882111488x^3 e^{x^2} + 36855741344619030234791936x^5 e^{x^2}$$

$$f^{(104)}(x) = 4 e^{x^2} + 13510798882111488x^2 e^{x^2} + 73711482689238060469583872x^4 e^{x^2}$$

$$f^{(105)}(x) = 8x e^{x^2} + 27021597764222976x^3 e^{x^2} + 73711482689238060469583872x^5 e^{x^2}$$

$$f^{(106)}(x) = 4 e^{x^2} + 27021597764222976x^2 e^{x^2} + 147422965378476120939167744x^4 e^{x^2}$$

$$f^{(107)}(x) = 8x e^{x^2} + 54043195528445952x^3 e^{x^2} + 147422965378476120939167744x^5 e^{x^2}$$

$$f^{(108)}(x) = 4 e^{x^2} + 54043195528445952x^2 e^{x^2} + 294845930756952241878335488x^4 e^{x^2}$$

$$f^{(109)}(x) = 8x e^{x^2} + 108086391056891904x^3 e^{x^2} + 294845930756952241878335488x^5 e^{x^2}$$

$$f^{(110)}(x) = 4 e^{x^2} + 108086391056891904x^2 e^{x^2} + 589691861513904483756670976x^4 e^{x^2}$$

$$f^{(111)}(x) = 8x e^{x^2} + 216172782113783808x^3 e^{x^2} + 589691861513904483756670976x^5 e^{x^2}$$

$$f^{(112)}(x) = 4 e^{x^2} + 216172782113783808x^2 e^{x^2} + 1179383723027808967513341952x^4 e^{x^2}$$

$$f^{(113)}(x) = 8x e^{x^2} + 432345564227567616x^3 e^{x^2} + 1179383723027808967513341952x^5 e^{x^2}$$

$$f^{(114)}(x) = 4 e^{x^2} + 432345564227567616x^2 e^{x^2} + 2358767446055617935026683904x^4 e^{x^2}$$

$$f^{(115)}(x) = 8x e^{x^2} + 864691128455135232x^3 e^{x^2} + 2358767446055617935026683904x^5 e^{x^2}$$

$$f^{(116)}(x) = 4 e^{x^2} + 864691128455135232x^2 e^{x^2} + 4717534892111235870053367808x^4 e^{x^2}$$

$$f^{(117)}(x) = 8x e^{x^2} + 1729382256910270464x^3 e^{x^2} + 4717534892111235870053367808x^5 e^{x^2}$$

$$f^{(118)}(x) = 4 e^{x^2} + 1729382256910270464x^2 e^{x^2} + 9435069784222471740106735616x^4 e^{x^2}$$

$$f^{(119)}(x) = 8x e^{x^2} + 3458764513820540928x^3 e^{x^2} + 9435069784222471740106735616x^5 e^{x^2}$$

$$f^{(120)}(x) = 4 e^{x^2} + 3458764513820540928x^2 e^{x^2} + 18870139568444943480213471232x^4 e^{x^2}$$

$$f^{(121)}(x) = 8x e^{x^2} + 6917529027641081856x^3 e^{x^2} + 18870139568444943480213471232x^5 e^{x^2}$$

$$f^{(122)}(x) = 4 e^{x^2} + 6917529027641081856x^2 e^{x^2} + 37740279136889886960426942464x^4 e^{x^2}$$

$$f^{(123)}(x) = 8x e^{x^2} + 13835058055282163712x^3 e^{x^2} + 37740279136889886960426942464x^5 e^{x^2}$$

$$f^{(124)}(x) = 4 e^{x^2} + 13835058055282163712x^2 e^{x^2} + 75480558273779773920853884928x^4 e^{x^2}$$

$$f^{(125)}(x) = 8x e^{x^2} + 27670116110564327424x^3 e^{x^2} + 75480558273779773920853884928x^5 e^{x^2}$$

$$f^{(126)}(x) = 4 e^{x^2} + 27670116110564327424x^2 e^{x^2} + 150961116547559547841707769856x^4 e^{x^2}$$

$$f^{(127)}(x) = 8x e^{x^2} + 55340232221128654848x^3 e^{x^2} + 150961116547559547841707769856x^5 e^{x^2}$$

$$f^{(128)}(x) = 4 e^{x^2} + 55340232221128654848x^2 e^{x^2} + 301922233095119095683415539712x^4 e^{x^2}$$

$$f^{(129)}(x) = 8x e^{x^2} + 110680464442257309696x^3 e^{x^2} + 301922233095119095683415539712x^5 e^{x^2}$$

$$f^{(130)}(x) = 4 e^{x^2} + 110680464442257309696x^2 e^{x^2} + 603844466190238191366831079424x^4 e^{x^2}$$

$$f^{(131)}(x) = 8x e^{x^2} + 221360928884514619392x^3 e^{x^2} + 603844466190238191366831079424x^5 e^{x^2}$$

$$f^{(132)}(x) = 4 e^{x^2} + 221360928884514619392x^2 e^{x^2} + 1207688932380476382733662158848x^4 e^{x^2}$$

$$f^{(133)}(x) = 8x e^{x^2} + 442721857769029238784x^3 e^{x^2} + 1207688932380476382733662158848x^5 e^{x^2}$$

$$f^{(134)}(x) = 4 e^{x^2} + 442721857769029238784x^2 e^{x^2} + 2415377864760952765467324317696x^4 e^{x^2}$$

$$f^{(135)}(x) = 8x e^{x^2} + 885443715538058477568x^3 e^{x^2} + 2415377864760952765467324317696x^5 e^{x^2}$$

$$f^{(136)}(x) = 4 e^{x^2} + 885443715538058477568x^2 e^{x^2} + 4830755729521905530934648635392x^4 e^{x^2}$$

$$f^{(137)}(x) = 8x e^{x^2} + 1770887431076116955136x^3 e^{x^2} + 4830755729521905530934648635392x^5 e^{x^2}$$

$$f^{(138)}(x) = 4 e^{x^2} + 1770887431076116955136x^2 e^{x^2} + 9661511459043811061869$$

Midterm I, November 25<sup>th</sup> 2008

(a) Absolute True Error = Exact Error - Approximated Error

$$E_t = \text{Exact Value} - \text{Approximated Value}$$

$$\text{Relative True Error} = \frac{\text{Absolute True Error}}{\text{Exact value}}$$

$$\% E_t = \frac{E_t}{\text{Exact value}}$$

$$\% E_t = \frac{E_t}{\text{Exact value}} \times 100$$

$$1) A = 10.147, \bar{A} = 10.159 \Rightarrow E_t = 10.147 - 10.159$$

$$\therefore E_t = -0.012 \Rightarrow |E_t| = 0.012$$

$$E_t = \frac{|E_t|}{10.147} = \frac{0.012}{10.147} \Rightarrow E_t = 0.001183$$

$$\% E_t = 0.1183 \%$$

$$2) A = 0.0047, \bar{A} = 0.0045, \text{ work with absolute values}$$

$$E_t = |0.0047 - 0.0045| \Rightarrow E_t = 0.0002$$

$$\% E_t = \frac{0.0002}{0.0047} \times 100 = \frac{0.02}{0.47} \times 100 \Rightarrow E_t = 4.255 \%$$

Even though the absolute error of (2) is less than that of (1) but the relative error of (2) is greater " " "

(1)

$$H = 0.671 \times 10^{-12} \quad \bar{H} = 0.669 \times 10^{-12}$$

$$E_t = |0.671 - 0.669| \times 10^{-12} = 0.002 \times 10^{-12}$$

$$\bar{E}_t = \frac{0.002 \times 10^{-12}}{0.671} \times 100 = 0.298$$

b) Find the Taylor Series of  $f(x) = e^{x^2}$

$$\therefore f(x_{i+1}) = f(x_i) + f'(x_i) \cdot h + \frac{f''(x_i) \cdot h^2}{2!} + \frac{f'''(x_i) \cdot h^3}{3!} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i) \cdot h^n}{n!}$$

$$f(x_{i+1}) = f(x_i) + \sum_{n=1}^{\infty} \frac{f^{(n)}(x_i) \cdot h^n}{n!}$$

$$Lx^2x^2e^{x^2} + 2e^{x^2}$$

$$\therefore f(x) = e^{x^2} \Rightarrow f'(x) = 2x e^{x^2}, \quad f''(x) = 4x^2 e^{x^2}$$

$$f^{(3)}(x) = 8x^3 e^{x^2}, \quad f^{(4)}(x) = 16x^4 e^{x^2}$$

$$\therefore f^{(n)}(x) = 2^n x^n e^{x^2} = (2x)^n e^{x^2}$$

$$f^{(0)}(x) = e^{x^2}$$

$$f(x_{i+1}) = \sum_{n=0}^{\infty} \frac{(2x)^n e^{x^2} \cdot h^n}{n!}$$

→ If you want to use MATLAB

$$= e^{x^2} + \sum_{n=1}^{\infty} \frac{(2x)^n e^{x^2} \cdot h^n}{n!}$$

→ using C program to avoid division by zero of factorial zero

$$c) T_n = \frac{2x_i \cdot T_{n-1} \cdot h}{n}$$

```
#include <stdio.h>
// #include <math.h>
// #include <conio.h>
main() {
```

```
int i=0, n; // n is the number of terms
```

```
float xi, x, fofx, Er; // *fofx is the value of f(xi+1)
```

```
float Tv, h;
```

```
Er is the relative True error */
```

```
/* Tv is the True value
```

```
h is step size */
```

```
Printf("\n Enter xi = "); scanf("%f", &xi);
```

```
Printf("\n Enter x = "); scanf("%f", &x);
```

```
Printf("\n Number of Terms = "); scanf("%d", &n);
```

```
h = fabs(x - xi);
```

```
Tv = exp(pow(x, 2)); // calculate the True value
```

```
Printf("\n h = %.5f \n True value = %.5f ", h, Tv);
```

```
fofx = exp(pow(xi, 2));
```

```
Er = 100 * fabs((Tv - fofx) / Tv);
```

```
Printf("\n Iteration f(x) R.T.E.P");
```

```
Printf("\n %d %.5f %.5f ", i, fofx, Er);
```

```
i = i + 1;
```

```
while(i <= n)
```

```
{
    xi
    fofx = (2 * xi) * fofx * h / i;
```

```
i = i + 1;
```

```
}
```

```
getche();
```

$$\underbrace{x^2 + 3y^2 - 1}_{h(x,y)} = \underbrace{(x-2)^2 + (y-1) + 4}_{g(x,y)}$$

At the intersection point  $\nabla h = \nabla g$

$$\therefore x^2 + 3y^2 - 1 - (x-2)^2 - (y-1) + 4 = 0$$

$$x^2 + 3y^2 - 1 - x^2 + 4x - 4 - y + 1 + 4 = 0$$

$$3y^2 - y + 4x = 0$$

Set  $f(x,y) = 3y^2 - y + 4x$

↳ Function of Two Variables

$$\frac{\partial f}{\partial x} = 4$$

i. guess it is out of the scope of the course

$$\frac{\partial f}{\partial y} = 6y - 1$$

$$x_{i+1} = x_i - \underline{\hspace{2cm}}$$

Q3

$$x_1 - x_2 + 2x_3 = 4$$

$$2x_1 + x_2 + 5x_3 = 5$$

$$-x_1 + 4x_2 + x_3 = -7$$

$$a) \begin{bmatrix} 1 & -1 & 2 & 4 \\ 2 & 1 & 5 & 5 \\ -1 & 4 & 1 & -7 \end{bmatrix} \begin{matrix} \uparrow \\ \leftarrow \\ \leftarrow \end{matrix} \Rightarrow \begin{bmatrix} 2 & 1 & 5 & 5 \\ 1 & -1 & 2 & 4 \\ -1 & 4 & 1 & -7 \end{bmatrix}$$

~~elimination~~ Forward elimination (backward substitution)

$$f_{12} = \frac{a_{12}}{a_{11}} = \frac{1}{2} = 0.5$$

$$f_{13} = \frac{-1}{2} = -0.5$$

$$\begin{aligned}
 R_2 - f_{12} \cdot R_1 & \quad (1 - \frac{1}{2} \times 2) \quad -1 - \frac{1}{2} \times 1 \quad 2 - \frac{1}{2} \times 5 \quad 4 - \frac{1}{2} \times 5 \\
 = R_2 - f_{12} \cdot R_1 & \quad -1 - (-0.5) \times 2 \quad 4 - (-\frac{1}{2} \times 1) \quad 1 - (-\frac{1}{2} \times 5) \quad -7 - (-\frac{1}{2} \times 5)
 \end{aligned}$$

$$\begin{bmatrix} 2 & 1 & 5 & 5 \\ 0 & -\frac{3}{2} & -\frac{1}{2} & \frac{3}{2} \\ 0 & \frac{9}{2} & \frac{7}{2} & -\frac{11}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 1 & 5 & 5 \\ 0 & -1.5 & -0.5 & 1.5 \\ 0 & 4.5 & 3.5 & -4.5 \end{bmatrix}$$

Eliminating  $x_2$  from  $R_3$

$$f_{23} = \frac{a_{32}}{a_{22}} = \frac{4.5}{-1.5} \Rightarrow f_{23} = -3$$

$$R_3 \leftarrow R_3 - f_{23} \cdot R_2 \Rightarrow \begin{bmatrix} 2 & 1 & 5 & 5 \\ 0 & -1.5 & -0.5 & 1.5 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

$$\therefore 2x_3 = 0 \Rightarrow x_3 = 0$$

$$-1.5x_2 - 0.5 \times 0 = 1.5 \Rightarrow x_2 = -1$$

$$2x_1 + 1(-1) + 5(0) = 5 \Rightarrow x_1 = 3$$

Substitute in the original eq<sub>s</sub>

$$x_1 - x_2 + 2x_3 = 4 \Rightarrow 3 - (-1) + 2(0) = 4$$

$$\begin{aligned}
 3 + 1 + 0 &= 4 \\
 4 &= 4 \quad \checkmark
 \end{aligned}$$

$$2x_1 + x_2 + 5x_3 = 2(3) - 1 + 5(0) = 5 = \text{L.H.S}$$

$$-x_1 + 4x_2 + x_3 = -(3) + 4(-1) + 0 = -7 = \text{L.H.S}$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 5 \\ -1 & 4 & 1 \end{vmatrix} = 1(1-20) + 1(2+5) + 2(8+1)$$

$$\therefore |\Delta = 6|, \Delta x = \begin{vmatrix} 4 & -1 & 2 \\ 5 & 1 & 5 \\ -7 & 4 & 1 \end{vmatrix} = 4(1-20) + 1(5+35) + 2(20+7)$$

$$= 4(-19) + 40 + 2(27)$$

$$= -76 + 40 + 54 = -36 + 54$$

$$= 18$$

$$\Delta x_1 = 18$$

$$\Delta y = \begin{vmatrix} 1 & 4 & 2 \\ 2 & 5 & 5 \\ -1 & -7 & 1 \end{vmatrix} = 1(5+35) - 4(2+5) + 2(-14+5)$$

$$= 40 - 4 \times 7 + 2(-9) = 40 - 28 - 18 = -6$$

$$\Delta x_2 = -6$$

$$\Delta x_3 = \begin{vmatrix} 1 & -1 & 4 \\ 2 & 1 & 5 \\ -1 & 4 & -7 \end{vmatrix} = 1(-7-20) + 1(-14+5) + 4(8+1)$$

$$= -27 - 9 + 36 = 0$$

$$\Delta x_3 = 0$$

$$x_1 = \frac{\Delta x_1}{\Delta}, x_2 = \frac{\Delta x_2}{\Delta}, x_3 = \frac{\Delta x_3}{\Delta}$$

$$x_1 = \frac{18}{6} = 3, x_2 = \frac{-6}{6} = -1, x_3 = \frac{0}{6} = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$$

The same result as in (b)



Group (B)

Alfateh University  
Electrical Engineering Department  
EE303 Numerical Analysis  
Mid-Term I

Answer all questions, Carry calculations to 3 decimal places, time allowed 1.5 hours

Q1-

a) Write a recursive expression in the form of  $T_{i+1} = (\dots) T_i$  for the following expression

$$\cosh x = 1 + \frac{1}{2} x^2 + \frac{1}{24} x^4 + \frac{1}{720} x^6 + \frac{1}{40320} x^8 + \dots$$

(3 Marks)

b) Starting at [57, 33] using the bisection method, what will be the relative error after 52 iterations.

(3 Marks)

c) Write a C program to perform bisection method for finding the root of nonlinear equation.

(4 Marks)

Q2 a) Using Newton's method, find the roots of the following two equations:

$$f_1(x, y) = 1 + x^2 - y^2 + e^x \cos(y) \text{ and } f_2(x, y) = 2xy + e^x \sin(y). \text{ Start with } x_0 = 1.0 \text{ and } y_0 = 4$$

Perform 3 iteration

(7 Marks)

b) Use the secant method to find the root of the following equation:

$$f(x) = 3x + \sin(x) - e^x, \text{ repeat until the error drops to below } 10^{-5}$$

(3 Marks)

Q3 a) Given  $A = \begin{bmatrix} 4 & -2 & 1 \\ -3 & -1 & 4 \\ 1 & -1 & 3 \end{bmatrix}$ , Find L and U of A and show that  $\det(A) = \det(L) \cdot \det(U)$

(6 Marks)

b) Given  $b = \begin{bmatrix} 15 \\ 8 \\ 13 \end{bmatrix}$ , solve the system  $Ax = b$  using Gaussian elimination method

(4 Marks)